MATHEMATICS (US)

Paper 0444/23
Paper 23 (Extended)

Key messages

To succeed in this paper candidates need to have completed full syllabus coverage, remember necessary formulae and definitions and show all working clearly. They should be encouraged to spend some time looking for the most efficient methods suitable in varying situations.

General comments

It is very important that candidates show their complete method in their working and to read each question carefully to ensure that the answer is given in the form requested.

Some candidates do not know the difference between some linked concepts, for example greatest common factor (GCF) and lowest common multiple (LCM), square/cube numbers and prime numbers, surface area and volume.

In reading scales on graphs it is clear that some candidates assume the scale is always going up in tenths. It is also important that any straight lines drawn are ruled. On graphs any corrections must be clearly shown and labelled.

Comments on specific questions

Question 1

This question was answered well although some responses showed little attention to the context and gave an answer of 4 or −4 by adding rather than subtracting.

Question 2

Many candidates showed the correct answer whilst others attempted to divide 32 by 5.

Question 3

27 for the cube number was usually correct in **part (a)** but in **part (b)** 57, 87 and very occasionally 77 were considered as prime.

Question 4

There was the obvious confusion between greatest common factor (GCF) and lowest common multiple (LCM) with some candidates giving 420 or 840 as the answer. Those who did give a factor as the answer sometimes gave 3 or 7.

Question 5

(a) The most common error was to round to the nearest integer, but many still gave two decimal places.

(b) There were very few candidates who did not know scientific form. However some wrote 18×10^{-4} or 0.18×10^{-2}.

Question 6

Most candidates could expand the brackets although a few did not simplify them leaving their answer as four terms. Common errors included thinking $x \times x = 2x$ and $3 \times 5 = 8$.

Question 7

Some candidates treated the gradient of the original line as 5 so they gave an answer of $-\frac{1}{5}$. Many did find the correct gradient of the original line but they did not know the rule for finding the gradient of the perpendicular line. In some responses there were attempts to write down the equation of a perpendicular line which was not requested and there was insufficient information to do this completely.

Question 8

Many candidates did not know the answers to these or a method to determine them. Some candidates did answer correctly with the most common incorrect answers being 30° and 60°.

Question 9

A very common incorrect answer was to give the volume of 315. Those who attempted area often worked out one of each of the three different areas, giving half of the required answer. Some gave just one area, either $9 \times 7 = 63$ or $7 \times 5 = 35$. Some knew that there were six areas but it was sometimes comprised of e.g. four of 9×7 with two of 7×5, giving a final answer of 322.

Question 10

Many responses started with $391 + n + n - 1$ but some omitted to divide this by 3 when equating to $5n$, so a common incorrect answer was 130. Those who did not use algebra found the question very challenging.

Question 11

(a) Most answers seen were correct although occasionally an incorrect common factor was selected, such as 2 or x.

(b) Again this was answered very well with the main error being incorrect signs inside the brackets $(x - 3)(y + 2)$.

Question 12

There were those who, having worked out the two sides 12 and 5, then found the gradient using either $5 \div 12$ or $12 \div 5$. Some found the sides of the triangle as 6 and 3 by adding the ordinates instead of subtracting them. Some then added $6 + 3$ to give an answer of 9.

Question 13

Those candidates who attempted to use $y = mx + c$ and form two simultaneous equations usually made errors and this method rarely led to the correct answer. The most successful method was to work out the gradient using difference in y ÷ difference in x. However some used difference in x ÷ difference in y, whilst others worked out difference in y incorrectly as $6 - 5 = 1$ rather than either 11 or –11. Those who wrote down the correct equation still made errors in making k the subject.

Question 14

(a) Many candidates wrote the expression as $\frac{1}{2}n$ or 2^{-n}. Some gave the next term rather than the nth term.

(b) Many candidates didn't recognise this sequence as powers of 5 so $5n$ and $5n^2$ were common answers. Some could not put the term-to-term rule of × 5 into an expression. Some gave the next term of 3125 as their answer.

Question 15

This question was answered very well. The main error was to perform the operations in the incorrect order, so adding before multiplying. Some did not know how to write two fractions with a common denominator and often just added the numerators and denominators together.

Question 16

(a) The most successful method was to find the cube root of 8 then convert to centimetres. Some attempted the other way so they would incorrectly write 800 cm^3 then try to cube root 800. Some others, after having reached 2 m, gave an answer of 20.

(b) Many candidates subtracted the powers of 10 so writing 5.1×10^0 or 5.1×10^1.

Question 17

(a) The reading was usually given accurately, although answers of 12.8 or 12.9 were sometimes seen. Some read the scale incorrectly as 12.84.

(b) The two points were usually accurately plotted although some candidates reversed the coordinates.

(c) The line of best fit was usually accurately drawn. Occasionally they were too steep and a few were not ruled.

Question 18

Many candidates confused $y = 3$ and $x = 3$ so instead of having two parallel vertical lines, there were two parallel horizontal lines. However the line $y = x + 3$ was often drawn correctly, although some plotted the line $y = x - 3$. The instruction was to shade the unwanted regions but many shaded the region they intended as their answer. Often the incorrect side of a line was identified, particularly the region $y \leqslant x + 3$ which many thought was the region above the line rather than below it.

Question 19

(a) This part was answered well with the most common error coming from those who raised 3 to the power 4 first and then subtracted 1, giving an answer of 80.

(b) This part was found to be more challenging with a often given as 6 and b often given as 1. Those who knew to halve the y-coefficient often found it challenging to work out the value of b.

Question 20

(a) The most common error was writing the expression as direct proportionality. In order to gain full credit candidates had to find the value of k and then write the expression using this value of k which many did not do.

(b) The biggest challenge for many candidates was rearranging the equation to make x, or at least x^2, the subject.

Question 21

(a) Candidates often gave the first and final number without giving the middle one, so for domain 2, 4 was a common answer and for range, 11, 17.

(b) Very few candidates used the correct term. Some used 'diminish' whilst others gave rotation or translation. Again very few gave the stretch factor or invariant line.

Question 22

The most common method was to find the volume of the cylinder and subtract the volume of the cone. This should then be divided by the area of the circular end to give the depth of water inside the cylinder. The main errors seen were either to use the wrong formula for either the volume or to add together the two correct volumes instead of subtracting them. A few candidates found the depth in the cylinder but then did not add on the depth in the cone, 6 cm.

Question 23

(a) Many correct responses were seen. The most common incorrect answers were 10 with 20 and 0 with 40.

(b) Fewer correct responses were seen in this part. The most common error was to divide each frequency by 10 so giving heights of 1.9, 1.6 and 0.8.

MATHEMATICS (US)

Paper 0444/43
Paper 43 (Extended)

Key messages

To achieve well in this paper, candidates need to be familiar with all aspects of the extended syllabus.

The recall and application of formulae and mathematical facts in varying situations is required as well as the ability to interpret situations mathematically and problem solve with unstructured questions.

Work should be clearly and concisely expressed with answers written to an appropriate accuracy.

Candidates should show full working with their answers to ensure that method marks are considered.

General comments

The standard of performance was generally good with most candidates attempting all questions. Some candidates showed working with stages that could be easily followed. In other cases, candidates omitted some stages or did not show calculations at all. For some candidates, improving presentation would help, as there were instances where candidates miscopied their own figures.

Candidates appeared to have sufficient time to complete the paper and any omissions were due to lack of familiarity with the topic or difficulty with the question rather than lack of time.

Some candidates lost marks by approximating values prior to the final answer. This was apparent, for example, in **Question 5** with cosine and sine values being written to just two figures. The requirement for accuracy to two decimal places in **Question 11** was often missed.

The topics that proved to be accessible were ratio, percentage decrease, compound interest, basic algebra techniques, change of subject, transformation geometry, plotting points and drawing curves, finding the mean of grouped data, interpretation of a cumulative frequency curve, using the sine rule and simple functions.

More challenging topics included average speed, tangent and gradient, area of a quadrilateral, bearings, angles in circles, sectors, surface areas and volumes of similar solids, generating and solving a quadratic equation, vector geometry and inverse functions.

Comments on specific questions

Question 1

(a) Many correct answers were seen in this part. Reflection in either the x-axis or the line $x = -1$ were the two most common incorrect answers. Only a few responses suggested two transformations, reflection in the x-axis followed by a translation.

(b) (i) Many correct rotations were seen. Some carried out a correct rotation about a wrong centre.

(ii) Although a majority of correct enlargements were seen, candidates were generally less successful in this part. Enlargements with scale factor 2 about a wrong centre were common. In a lot of cases candidates drew the enlargement so that one of the vertices of the trapezium was placed at $(-7, 0)$.

(iii) Although some candidates had a good understanding of a stretch many others struggled to make any progress. This was reflected by the high number of candidates that did not attempt the question. Incorrect responses often included a stretch with the *x*-axis invariant. In some cases, a shape other than *A* was stretched.

Question 2

(a) (i) Many correct answers were seen from candidates of all abilities. Some did not read the question carefully enough and calculated the number of males. A few mistakenly divided the 342 by 11 instead of 3.

(ii) Although the simple approach would have been to find 3 parts as a percentage of 11 parts, many chose to use their calculated answer from the previous part. Many did so correctly but those with an incorrect answer did not achieve a correct answer to this part. Several candidates did not give an answer correct to three significant figures or more.

(b) The vast majority of candidates had a good understanding of percentage discount and many correct answers were seen. Roughly equal numbers opted to calculate 85% of the price as did 15% of the price. Some who calculated the discount then forgot to subtract it from the price or did so incorrectly. A small number treated this as a reverse percentage question.

(c) Although this proved more challenging, a significant number understood the method for calculating a reverse percentage. Those that recognised that 79.50 was 106% of the original number of candidates almost always went on to obtain the correct answer. Some had not taken enough care when reading the question and answers of 75, the previous cost of membership, were common. Reducing, or increasing, 79.50 by 6% was a common error.

(d) This proved to be more demanding than the other parts and this was reflected in the number of incorrect responses. More able candidates generally reached the correct answer. Calculating the total time for the journey produced many errors with the calculated time of 2.25 hours added to the given time of 2 hours 24 minutes to give an incorrect total, usually one of 4 hours 49 minutes or 4.49 hours. Many of the less able candidates calculated the speed for the first part of the journey and found the average of the two speeds.

(e) Candidates had a good understanding of compound interest with most using the formula to obtain the answer. Some carried out year-on-year calculations which sometimes led to inaccurate answers either due to premature rounding or use of an incorrect number of years. A number of candidates treated the question as if it involved simple interest.

(f) A good majority of candidates carried out the substitution correctly and went on to obtain the correct answer. The most common error was giving an answer to the wrong degree of accuracy.

Question 3

(a) (i) Candidates were asked to show $3a + 5b = 170$ and $a + 2b + 3b + 10 + 2a = 180$ was the required starting point. Many correct answers were seen, although several started from $3a + 5b + 10 = 180$.

(ii) Again, many correct answers were seen with some starting from a partially simplified expression as in **part (a)(i)**.

(iii) Candidates had a good understanding of simultaneous equations and most were able to solve them correctly. The elimination method was commonly used but a significant number used a substitution method which tended to produce more arithmetic slips due to the introduction of fractions.

(iv) When the answer to **part (a)(iii)** was correct it was common to see a correct answer for the smallest angle. Some did not appreciate what was required and simply identified the smallest angle by giving an answer of $2a$.

(b) Almost all candidates were able to solve this equation. Most errors stemmed from incorrectly dealing with $–12 + 3$, usually given as 9.

(c) Most candidates understood the steps involved in rearranging the given equation and many correct answers were seen. Dealing with the negatives produced most of the errors; some were just lost in the rearrangement while others did not change when moving terms across the equation.

Rearranging $8x - 2y = 5x - 3$ to $2y = -3x - 3$ and rearranging $-2y = -3x - 3$ to $y = \frac{-3x - 3}{2}$ were just two of the typical errors.

(d) More able candidates had no difficulty in working with the indices and most achieved the correct result. Less able candidates were more likely to apply the rules of indices incorrectly. Most errors involved the coefficient of 27 and it was common to see 27 multiplied by $\frac{2}{3}$. In a few cases, candidates applied only part of the power, either squaring or cube rooting and omitting the other.

(e) Those candidates that realised that factorisation was required to simplify the fraction usually did so without error. Where errors were seen it was common to see $(x - 5)^2$ used for the denominator. Many of the less able candidates did not appreciate the need for factorisation and it was common to see the x^2 terms being cancelled.

Question 4

(a) The vast majority of candidates completed the table correctly. Calculating the value of y as 3 for $x = -1$ was the most common error.

(b) Many candidates plotted the points correctly and drew an acceptable curve. Plotting (–3, –3) at (–3, 3) was sometimes seen but missing plots or incorrectly plotting at (1.5, –1.9) and (2.5, 9.4) were more common. Very few joined up the points with ruled lines and, if seen, usually occurred in the first and last segments of the curve.

(c) A good number of correct solutions were seen. Some candidates did not appear to understand that they were looking for the points of intersection of the curve with the x-axis. Many of the incorrect responses gave solutions that were not close to their points of intersection, such as –1, 2 and 5. A higher proportion of candidates made no attempt.

(d) A majority of candidates were able to draw an accurate tangent and many went on to calculate its gradient correctly. In a lot of cases, tangents were not touching the curve, crossed the curve or were drawn at the wrong point. Misreading the scales was a cause of many of the errors in calculating the changes in x and y. Numerical slips were also seen, largely because of the negative values used. Some less able candidates made no attempt at this part.

(e) Only a minority of candidates gave the correct value of the integer k. Writing $k = 21$ was by far the most common error with candidates seemingly ignoring the fact that the equation had to have three solutions. Several non-integer solutions were seen.

Question 5

(a) Although good solutions were seen many candidates found this challenging as there was no diagram showing the post. Many drew sketches but not all were labelled correctly. The majority of correct responses used the tangent ratio with a small number using the sine rule. A high proportion of candidates made no attempt.

(b) A majority of candidates showed their working clearly and obtained the correct value for the angle. Some started with $107^2 = 132^2 + 158^2 - 2 \times 132 \times 158 \times \cos A$ and then rearranged it incorrectly trying to obtain an expression for $\cos A$. Others had a correct statement of the cosine rule for an angle other than A and some stated the cosine rule incorrectly. A small number gave the final answer as an integer.

(c) Candidates were more successful when using the sine rule and a greater number of correct responses were seen. As in the previous part, errors were seen when rearranging the rule to obtain an expression for $\sin CAD$.

(d) Fully correct answers were in the minority although some candidates earned partial credit for a correct method for the area using their incorrect angles from the previous two parts. Those candidates using $\frac{1}{2}ab\sin C$ were generally more successful although some slipped up by using angle *CAD* instead of angle *ACD*. Some candidates used the sine rule to calculate *AD* and then used the angle *CAD*. Several candidates attempted to calculate a correct perpendicular height for each triangle but did so by assuming the height bisected the base. A high proportion of candidates made no attempt at a response.

(e) This part proved challenging and only a minority found the correct bearing. Some candidates found the reverse bearing from *C* to *A*. Many of the other attempts showed no obvious pattern to the errors that were seen.

Question 6

(a) (i) The median was stated correctly by the vast majority of candidates. Misinterpreting the scale on the horizontal axis was common and 46 was the common error. A few candidates gave the cumulative frequency of 30 for the median value.

(ii) Again, many correct answers were seen. Misreading the scale and giving the cumulative frequency were the common errors again.

(iii) Although many candidates obtained the correct value for the interquartile range, fewer correct answers were seen than in the previous parts. As well as similar errors to those in the previous two parts, some gave incorrect answers such as 36 – 62 while others gave the value of the lower quartile.

(b) There were many incorrect responses, most of which did not involve a comparison in context. Candidates were expected to comment about the greater variation in the distances travelled by the group of women. Some typical incorrect comments included ‘female cyclists have a larger distribution’ and ‘females travelled further than males’. Other comments made reference to the strength or fitness of the female cyclists compared with the male cyclists.

(c) Many candidates had no difficulty in finding the probability. Common errors included the probability that the distance travelled was less than 50 km and, in some cases, candidates misread the vertical scale.

(d) (i) Completing the frequency table was answered well. Occasionally candidates misread the scale and this led to values close to the correct values. In some cases, the values were a long way out, giving total frequencies a lot different from 60.

(ii) Calculating the mean of grouped data was well answered with most candidates showing a clear and accurate method. If the answer to **part (d)(i)** was correct then the working usually led to the correct mean. A small minority of incorrect answers resulted from the use of either the upper bounds of the intervals or use of the interval widths.

Question 7

(a) (i) Only a minority of candidates were able to show angle $AOC = 104°$ with fully correct reasons. Many showed a jumble of calculations with no angles clearly defined, either by labelling or by marking on the diagram. Some did identify the angles but their reasons were frequently incomplete or incorrect. Some incorrect statements included ‘angles in a quadrilateral add to 180°’, ‘angle at the centre is twice the angle at the circle’.

(ii) Candidates were far more successful in this part and a majority of candidates obtained the correct answer. There was a variety of methods available to choose from and all of them were seen. Errors arose when candidates made incorrect assumptions, typically that angle *BAD* and angle *BCD* were right-angles.

(iii) This part proved more demanding and only a minority of candidates obtained the correct angle. Many did not use the fact that angle *ABD* and angle *ACD* were subtended by the same arc (or chord). Instead, some successfully worked out the angle by considering angles from various triangles and quadrilaterals. A common error was to assume that *BD* bisected angle *ABC* giving 26° as a common incorrect answer. A very high proportion of candidates made no attempt at a response.

(iv) Very few candidates were able to calculate the correct perimeter. Some attempts used the area formula instead of the formula for circumference. In a lot of cases, it seemed that the word 'sector' had been overlooked or ignored as many candidates attempted to find the perimeter of the quadrilateral *OADC*. A very high proportion of candidates made no attempt at a response.

(b) (i) More able candidates could show a clear method and almost always obtained the correct cross-sectional area. Less able candidates clearly had some idea of the different scale factors but frequently applied the wrong powers when going from volume to length to area. Common incorrect working included $36 \times \frac{2187}{648}$ and $36 \times \sqrt[3]{\frac{2187}{648}}$.

(ii) Candidates were far more successful in this part. Good responses showed clearly presented working that often led to the correct radius. Having reached $r^3 = 522.1$ some continued by taking the square root. Less able candidates could make a start but some made errors in rearranging the formula to find r.

Question 8

(a) (i) Some correct answers were seen. Common errors included the addition of the two probabilities, just giving the answer as $\frac{4}{6}$ and a few used with replacement.

(ii) Those that realised the two events in **part (a)** were mutually exclusive almost always obtained the correct answer. A significant number of candidates misinterpreted 'not both red' as 'both not red' and an answer of $\frac{1}{15}$ was seen.

(b) Only the more able candidates could make much progress and fully correct responses were in the minority. Some candidates attempted a tree diagram but, in many cases, the diagram did not reflect the events described in the question. A few candidates realised that obtaining three blues was mutually exclusive with the required outcome and used this approach successfully. The remaining correct responses came from those that identified the three relevant outcomes as G, BG and BBG. It was common to see candidates ignoring the probabilities of picking a blue and concentrating on the probability of a green on the first, second and third picks. This often led to the probabilities $\frac{2}{7}$, $\frac{2}{6}$ and $\frac{2}{5}$ being multiplied and in some cases added.

Question 9

(a) (i) Many of the more able candidates could give the correct vector. Others did not allow for directions and $8\mathbf{b} + 4\mathbf{a}$ was a common incorrect answer. A few candidates gave the reverse vector. A significant number of candidates made no attempt at a response.

(ii) Candidates were more successful in this part. Some did not simplify their answer as required and seeing $\frac{3}{4}8\mathbf{b}$ was quite common. A significant number of candidates made no attempt at a response.

(iii) A small majority of candidates were able to give the correct vector. Again, some did not allow for directions and $6\mathbf{b} + 2\mathbf{a}$ was a common incorrect answer. A high proportion of candidates made no attempt at a response.

(b) Candidates who were successful in the earlier parts tended to be more successful in finding a vector for *B* to *C* which usually led to the correct ratio. As in previous parts, some candidates did not allow for directions of vectors. When answers were incorrect it was difficult to award part marks as candidates did not clearly identify the method or routes they were using. A very high proportion of candidates made no attempt at a response.

Question 10

(a) (i) Almost all candidates calculated the value of the function correctly.

(ii) Many correct responses were seen in this part also. Most candidates evaluated g(3) as their first step and then substituted its value into the function h(x). Common errors usually involved answers of 0 or 3. Few attempted the composite function hg(x) as a first step and when seen it was often incorrectly given as $3^x(9-x^2)$.

(iii) Only a minority of correct answers were seen. The most common error was $9 - 2x^2$.

(iv) The composition of two functions, fg(x), was more challenging but a majority of candidates answered this part well. Errors frequently occurred in simplifying $2(9 - x^2) - 3$, either forgetting to multiply the square term by 2 or dealing with the negative terms incorrectly. Some gave their final answer as $2x^2 - 15$. Many of the less able candidates took the composition to be a product of the two functions.

(b) Finding the inverse of a linear function was well answered. Sign errors were common when rearranging the terms, particularly from less able candidates. A number of candidates did not earn full credit as they left an otherwise correct answer in terms of y. Some others gave the answer as the reciprocal of $2x - 3$.

(c) This was generally well answered with most candidates expanding $5(2x - 3)$ and following through to a correct answer. Common errors usually involved either incorrect expansion of the bracket, usually $10x - 3$, or occasional slips in the rearrangement.

(d) Candidates found this part challenging and only a minority were able to obtain the correct solutions. Most candidates opted to expand and simplify their terms whilst some opted to rearrange the equation to the form $(2x - 3)^2 = 25$ and then square root. This left straightforward algebra to find the solutions. Expansion and simplification of $9 - (2x - 3)^2$ produced many errors as it often appeared as $9 - 4x^2 - 12x + 9$ and in some cases as $9 - 4x^2 - 9$.

(e) Success in this question was largely dependent on the algebraic step of moving from $h^{-1}(x) = -2$ to $x = h(-2)$. This was rarely seen and only a small minority obtained the correct answer. The log function was used in a very small number of cases with limited success. Several candidates treated the inverse function as the reciprocal function. A high proportion of candidates made no attempt at a response.

Question 11

This proved to be challenging for many candidates and this was reflected by a minority of correct responses. Most recognised that they needed to eliminate the fractions but were not able to do so correctly. Common errors at this stage included multiplying only one side of the equation by the common denominator. Some opted to write the two terms on the left as a single fraction, quite often successfully, before cross-multiplying. Expansion of the resulting brackets often caused difficulty and it was common to see $x(x + 1)$ expanded as $x^2 + 1$. Rearrangement of the resulting terms often included errors with the signs and reaching the correct three-term quadratic equation was rare. When candidates did reach a three-term quadratic fewer errors were seen in their attempts to solve it. Some of those with a completely correct method tended to give both answers correct to three significant figures, perhaps having forgotten the instruction asking for two decimal places.